Barcelona Mathematical Days 2017

Congrés de la Societat Catalana de Matemàtiques

Institut d'Estudis Catalans

Barcelona

April 27–28, 2017

Schedule and Abstracts

Contents

General Schedule

Thursday, April 27

08:15 – 08:45	**Registration**	
08:45 – 09:00	**Opening**	Room Prat de la Riba
09:00 – 10:00	**Plenary talk:** Sara van de Geer	Room Prat de la Riba
10:00 – 10:30	**Coffee break**	
10:30 – 13:00	**Thematic sessions:**	
	Mathematics in Space Science	Room Pere Coromines
	Modularity and Diophantine Equations: Exploring Wiles' Universe	Room Nicolau d'Olwer
	New Perspectives in PDEs and Applications	Room Pi i Sunyer
15:00 – 16:00	**Plenary talk:** Samir Siksek	Room Prat de la Riba
16:00 – 16:30	**Coffee break**	
16:30 – 19:00	**Thematic sessions:**	
	Algebra, Geometry and Biology	Room Pere Coromines
	In and Around the Mapping Class Group	Room Nicolau d'Olwer
	Progress in Transport Phenomena	Room Pi i Sunyer
20:00 –	**Conference dinner**	

Friday, April 28

09:00 – 10:00	**Plenary talk:** Marino Arroyo	Room Prat de la Riba
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10:30 – 13:00	**Thematic sessions:**	
	Mathematics in Space Science	Room Pere Coromines
	Modularity and Diophantine Equations: Exploring Wiles' Universe	Room Nicolau d'Olwer
	New Perspectives in PDEs and Applications	Room Pi i Sunyer
15:00 – 16:00	**Plenary talk:** Daniel Peralta Salas	Room Prat de la Riba
16:00 – 16:30	**Coffee break**	
16:30 – 19:00	**Thematic sessions:**	
	Algebra, Geometry and Biology	Room Pere Coromines
	In and Around the Mapping Class Group	Room Nicolau d'Olwer
	Progress in Transport Phenomena	Room Pi i Sunyer

Plenary Lectures

Schedule

Thursday 09:00 – 10:00, Room Prat de la Riba

Sara van de Geer

ETH Zürich

Concentration in high-dimensional statistics

Thursday 15:00 – 16:00, Room Prat de la Riba

Samir Siksek

University of Warwick

Which integers are sums of seven cubes?

Friday 09:00 – 10:00, Room Prat de la Riba

Marino Arroyo

Universitat Politècnica de Catalunya and Institute for Bioengineering of Catalonia

Mathematical modeling and simulation of the cell envelop, an active and adaptable biological interface

Friday 15:00 – 16:00, Room Prat de la Riba

Daniel Peralta-Salas

Instituto de Ciencias Matemáticas, Madrid

A problem of Berry and knotted zeros in the eigenfunctions of the harmonic oscillator

CONCENTRATION IN HIGH-DIMENSIONAL STATISTICS

SARA VAN DE GEER

ETH Zürich

High-dimensional statistics is about models where one has more parameters p than observations n. The theory contains a high concentration of challenging mathematical issues. We will encounter convex analysis, random matrix theory, approximation theory and, last but not least, concentration of measure. We illustrate this for linear regression and also briefly discuss other models.

Suppose one observes an n-dimensional Gaussian vector Y of the form

$$Y = f^0 + \epsilon,$$

with f^0 an unknown mean vector and ϵ standard Gaussian noise. Consider a given $n \times p$ design matrix X, with $p > n$, and the estimator

$$\hat{\beta} := \arg\min_{\beta \in \mathbb{R}^p} \left\{ \underbrace{\|Y - X\beta\|_2^2}_{\text{least squares loss}} + \underbrace{2\lambda\|\beta\|_1}_{\text{regularization penalty}} \right\}$$

where $\lambda > 0$ is a given tuning parameter. This estimator, called "Lasso" (Tibshirani (1996)), is extremely popular in high-dimensional regression. The penalty $\beta \mapsto 2\lambda\|\beta\|_1$ regularizes the problem: it ensures that certain entries in the vector $\hat{\beta}$ are set to zero.

The theoretical properties of the Lasso are well understood. We will present some recent further refinements of this theory.

Consider the minimizer of the noiseless problem

$$\beta^* := \arg\min_{\beta \in \mathbb{R}^p} \left\{ \|f^0 - X\beta\|_2^2 + 2\lambda \|\beta\|_1 \right\}.$$

Let $\hat{f} := X\hat{\beta}$ and $f^* := X\beta^*$.

We are interested in the behaviour of the "approximation error" $\|f^* - f^0\|_2$ and "estimation error" $\|\hat{f} - f^*\|_2$. Note that the approximation error is a deterministic quantity that can be studied using approximation theory. Clearly, if the tuning parameter λ is small this error will be small too. The estimation error is random and is typically large for small λ.

To deal with the estimation error one can apply concentration of measure. It is shown in van de Geer and Wainwright (2016) and Bellec and Tsybakov (2016) that $\epsilon \mapsto \|\hat{f} - f^*\|_2$ is Lipschitz. Hence by concentration of measure, $\|\hat{f} - f^*\|_2$ concentrates around its median, m^* say. A bound for the median m^* is as follows. The subdifferential of $\beta \mapsto \|\beta\|_1$ is equal to

$$\partial\|\beta\|_1 := \{z \in \mathbb{R}^p : \ \|z\|_\infty \leq 1, \ \beta^T z = \|\beta\|_1\}.$$

The vector β^* satisfies the Karush-Kuhn-Tucker conditions

$$X^T(X\beta^* - f^0) + \lambda z^* = 0$$

where $z^* \in \partial\|\beta^*\|_1$. For any $S \subset \{1, \ldots, p\}$ let z^*_{-S} be the vector z^* restricted to the complement of the set S.

Theorem *Let $S \subset \{1, \ldots, p\}$ be a subset of the variables such that $\|z^*_{-S}\|_\infty < 1$. Let $\lambda(1 - \|z^*_{-S}\|_\infty) \geq \sqrt{2n\log(4p)} + \sqrt{2n\log(2)}$. Then have*

$$m^* \leq \sqrt{|S|} + \sqrt{2\log(2)}.$$

The flavour of this result is that the squared estimation error is roughly the number of variables (columns of X) needed to approximate the

signal f^0. In other words, the estimator adapts to the sparsity of the approximation f^* of f^0.

Moreover, under certain conditions on the design X one can show that $\sqrt{|S|}$ is of small order $\|f^* - f^0\|_2$. Thus, typically, the approximation error dominates the estimation error. Applying random matrix theory, the design conditions are met with large probability when X is a random matrix with independent rows from an appropriate distribution.

A bound for the approximation error $\|f^* - f^0\|_2$ can be established using convex analysis. Here occurs a geometric quantity that can be thought of as an ℓ_1 version of canonical correlation. We show in some examples that the obtained bound is tight.

Finally, we present some extensions to other norms and loss functions for non-Gaussian models.

References:

P.C. Bellec and A.B. Tsybakov. Bounds on the prediction error of penalized least squares estimators with convex penalty, 2016. arXiv:1609.06675v1.

R. Tibshirani. Regression analysis and selection via the Lasso. *Journal of the Royal Statistical Society Series B*, 58:267–288, 1996.

S. van de Geer and M. Wainwright. On concentration for (regularized) empirical risk minimization, 2016. arXiv:1512.00677v2, to appear in *Sankhya*.

WHICH INTEGERS ARE SUMS OF SEVEN CUBES?

SAMIR SIKSEK
University of Warwick

In 1851, Carl Jacobi made the experimental observation that all integers are sums of seven non-negative cubes, with precisely 17 exceptions, the largest of which is 454. Building on previous work by Maillet, Landau, Dickson, Linnik, Watson, Bombieri, Ramaré, Elkies and many others, we complete the proof of Jacobi's observation.

MATHEMATICAL MODELING AND SIMULATION OF THE CELL ENVELOP, AN ACTIVE AND ADAPTABLE BIOLOGICAL INTERFACE

MARINO ARROYO

Universitat Politècnica de Catalunya and Institute for Bioengineering of Catalonia

The cell envelop is an active and adaptable interface that plays a fundamental role in many biological events. To name a few, it determines cell shape and mechanics, controls the way cells adhere to each other, drives cell division and motility, and controls the structure and remodeling of multicellular tissues. Fundamental mechanical and chemical determinants of its biological function can be described with mathematical models involving partial differential equations on curved surfaces coupled with the geometric evolution laws for those surfaces. In this talk, I will describe instances of such models coupling chemistry, elasticity, hydrodynamics and active force generation, their numerical discretization, and how they can be used to connect with experiments and understand various phenomena in cell mechanobiology.

A PROBLEM OF BERRY AND KNOTTED ZEROS IN THE EIGENFUNCTIONS OF THE HARMONIC OSCILLATOR

DANIEL PERALTA-SALAS
Instituto de Ciencias Matemáticas, Madrid

In 2001, motivated by finding eigenfunctions of the hydrogen atom whose nodal sets form torus knots, the theoretical physicist Michael Berry conjectured the same was true for the harmonic oscillator and asked whether any finite link could be realized as the nodal set of an eigenfunction to some quantum system. In this talk we give a proof that answers both these questions in the affirmative. Specifically, we show that any finite link in Euclidean space can be realized (up to a global diffeomorphism) as the union of connected components of the nodal set of a harmonic oscillator eigenfunction. We will see that the high energy asymptotics of the harmonic oscillator's eigenfunctions and the wealth of different solutions to the Helmholtz equation play key roles in this proof. Time permitting, I will also show that an analogous result holds for eigenfunctions of the hydrogen atom, which matches Berry's original setting. This is joint work with Alberto Enciso and David Hartley.

Thematic Session
Algebra, Geometry and Biology

Organized by

Marta Casanellas
Universitat Politècnica de Catalunya

Carsten Wiuf
Københavns Universitet

Schedule

Thursday 16:30 – 17:15, Room Pere Coromines

Dorothy Buck

Imperial College London

Knotted DNA; Mathematical Models and Biological Consequence

Thursday 17:20 – 18:05, Room Pere Coromines

Nina Otter

Mathematical Institute, University of Oxford

The phylogenetic operad

Thursday 18:10 – 12:55, Room Pere Coromines

Piotr Zwiernik

Universitat Pompeu Fabra, Barcelona

The correlation space of Gaussian latent tree models and model selection without fitting

Friday 16:30 – 17:15, Room Pere Coromines

Jesús Fernández-Sánchez

Universitat Politècnica de Catalunya, Barcelona

The embedding problem for Markov matrices of evolutionary models

Friday 17:20 – 18:05, Room Pere Coromines

Elisenda Feliu

Københavns Universitet

Algebraic parametrizations in biochemical reaction networks

Friday 18:10 – 18:55, Room Pere Coromines

Matteo Ruffini

Universitat Politècnica de Catalunya, Barcelona

Clustering patients with Tensor Decomposition

KNOTTED DNA; MATHEMATICAL MODELS AND BIOLOGICAL CONSEQUENCES

DOROTHY BUCK

Imperial College London

DNA, like any other long piece of string packed into a small space, would become highly knotted and tangled if there were no mechanisms to both keep it organised, and to untangle any knots that do arise. Every living organism has developed mechanisms to control DNA knotting in its cells, and many antibiotics and chemotherapeutics act by disturbing this control. This talk will give an overview of some of the topological methods that we utilise to model DNA knotting, and how the answers from these models aid experimentalists. (No prior biological knowledge needed.)

THE PHYLOGENETIC OPERAD

NINA OTTER
University of Oxford

Phylogenetics is concerned with the study of evolutionary relationships among species or genes. These relationships are usually represented by metric trees called phylogenetic trees. In 2001 Billera, Holmes and Vogtmann introduced a space that parametrizes the set of all phylogenetic trees with a fixed set of leaf labels, and since then a lot of research has been done to understand the combinatorics and geometry of this space to develop suitable statistical methods. In this talk I will present joint work with John Baez that relates the space of phylogenetic trees to a certain operad, which we call the phylogenetic operad. I will first introduce the space of phylogenetic trees, and talk about some of its combinatorial and geometric properties. Then I will give a gentle introduction to operads and describe two results: the space of phylogenetic trees with n leaves is homeomorphic to the space of n-ary operations of the phylogenetic operad, and Markov processes used in phylogenetics give coalgebras over this operad.

THE CORRELATION SPACE OF GAUSSIAN LATENT TREE MODELS AND MODEL SELECTION WITHOUT FITTING

PIOTR ZWIERNIK
Universitat Pompeu Fabra

In phylogenetics and lynguistics latent tree models are used to model evolutionary processes. Model selection procedures are employed to choose the best tree fitting the data. However, deciding if the tree hypothesis is consistent with the data is typically hard. We provide the full semialgebraic description of Gaussian latent tree models and link them to phylogenetic oranges. We then use this geometric description to propose a quick and robust way of choosing the best tree, or, of testing the tree hypothesis.

This is joint work with John Aston, Nat Shiers, and Jim Smith.

THE EMBEDDING PROBLEM FOR MARKOV MATRICES OF EVOLUTIONARY MODELS

JESÚS FERNÁNDEZ-SÁNCHEZ
Universitat Politècnica de Catalunya, Barcelona

Continuous-time Markov processes are often used to model evolution on nucleotide sequences. The resulting models consider nucleotide substitution matrices that are the exponential of rate matrices and assume some extra restrictions in contrast to biological reality. A more general approach considers models where the parameters are given by the substitution probabilities between nucleotides. The understanding of the connection between these two approaches is fundamental for modelling evolution as it has practical and theoretical consequences, as the identifiability of rates from experimental biological data. In this talk, we will give a description of the embedding problem, and see some new results about embeddability when restricted to well-known evolutionary models.

This is a joint work with Marta Casanellas and Jordi Roca-Lacostena.

ALGEBRAIC PARAMETRIZATIONS IN BIOCHEMICAL REACTION NETWORKS

ELISENDA FELIU
University of Copenhagen

It is widespread in biology to model the time evolution of species concentrations in reaction networks with systems of ordinary differential equations. Current models in systems biology employ polynomial dynamical systems and, as a consequence, the steady states of the model are the zeros of a system of polynomial equations. These systems depend often on unknown parameters and, further, only positive solutions have a physical meaning. Therefore, it is of interest to study the positive zeros of a family of polynomials with unknown coefficients.

Recent work has highlighted the use of algebraic parameterisations to infer properties on the number of steady states, their stability and how the steady states depend on the unknown parameters.

In the talk I will briefly present the framework of the theory of reaction networks and give an overview of questions that are relevant for systems biology. I will then focus on my recent work on algebraic parameterisations: how to find them and what they can be used for.

The presented results are in collaboration with Carsten Conradi, Janne Kool, Maya Mincheva, Meritxell Sáez, Angélica Torres and Carsten Wiuf.

CLUSTERING PATIENTS WITH TENSOR DECOMPOSITION

MATTEO RUFFINI

Universitat Politècnica de Catalunya, Barcelona

Clustering patients in groups with similar clinical profiles is a strategic activity for a modern healthcare systems. This, typically requires to deal with high-dimensional categorical data, a framework where most of the traditional clustering algorithms perform poorly. A viable alternative is to use a clustering procedure based on generative latent variable models, a task that requires fitting the parameters of the models in question. In recent times, learning the parameters of latent variable models has become a popular research branch, due to the nice tensor structure of their low-order observable moments. Methods based on tensor decomposition have emerged as effective and efficient alternatives to the existing heuristics and their applications are rapidly growing.

In this talk, we will see how tensor decomposition can be used to fit the parameters of latent variable models, focusing on some special cases of unlabeled mixture models and on how they can be used as a clustering tool. We will then see the application of this clustering procedure to medical data, using it to automatically generate clusters of patients with homogeneous clinical profiles.

Thematic Session

In and Around the Mapping Class Group

Organized by

Wolfgang Pitsch
Universitat Autònoma de Barcelona

Juan Souto
Université de Rennes 1

Schedule

Thursday 16:30 – 17:15, Room Nicolau d'Olwer

Rinat Kahaev

Université de Genève

Mapping class group representations in the Hilbert space of square integrable functions over locally compact abelian groups

Thursday 17:20 – 18:05, Room Nicolau d'Olwer

Javier Aramayona

Universidad Autónoma de Madrid

Finitely-presented big mapping class groups

Thursday 18:10 – 12:55, Room Nicolau d'Olwer

Hugo Parlier

University of Fribourg

Interrogating length spectra and quantifying isospectral finiteness

Friday 16:30 – 17:15, Room Nicolau d'Olwer

Marithania Silvero

University of Warsaw

On Khovanov homology of links

Friday 17:20 – 18:05, Room Nicolau d'Olwer

Julien Marché

Université Pierre et Marie Curie, Paris

A Jones polynomial for curves on surfaces

Friday 18:10 – 18:55, Room Nicolau d'Olwer

Ken Bromberg

University of Utah

Convex co-compact subgroups of the mapping class group

MAPPING CLASS GROUP REPRESENTATIONS IN THE HILBERT SPACE OF SQUARE INTEGRABLE FUNCTIONS OVER LOCALLY COMPACT ABELIAN GROUPS

RINAT KASHAEV
Université de Genève

Let S be an oriented punctured surface of finite type of negative Euler characteristic. By using the combinatorics of ideal triangulations of S and the notion of a quantum dilogarithm over a self-dual locally compact abelian group A, I will describe a construction of a unitary projective representation of the mapping class group of S in the Hilbert space of square integrable functions over A.

FINITELY-PRESENTED BIG MAPPING CLASS GROUPS

JAVIER ARAMAYONA
Universidad Autónoma de Madrid

(Joint work with Louis Funar.) In this talk, we will define the "asymptotically rigid" mapping class group B_g of genus g. The group B_g is a subgroup of the mapping class group of a certain infinite-type surface, and contains the mapping class group of every compact surface of genus $\leq g$ with non-empty boundary.

The group B_g turns out to be finitely presented, and its k-th homology group coincides with the k-th stable homology of the mapping class group of genus g, for k small enough with respect to g.

We then proceed to prove that the family of groups so obtained enjoys a number of properties analogous to known results for finite-type mapping class groups. More concretely, there are no (weakly) injective maps $B_h \to B_g$ if $g < h$, every automorphism of B_g is geometric, and every homomorphism from a higher-rank lattice to B_g has finite image.

In addition to the translation of classical results about finite-type mapping class groups, we will show that B_g is not linear, and does not have Kazhdan's Property (T).

INTERROGATING LENGTH SPECTRA AND QUANTIFYING ISOSPECTRAL FINITENESS

HUGO PARLIER
University of Fribourg

Associated to a closed hyperbolic surface is its length spectrum, the set of the lengths of all of its closed geodesics. Two surfaces are said to be isospectral if they share the same length spectrum. There are different methods to produce surfaces that are isospectral but not isometric, the most successful one based on a technique introduced by Sunada.

The talk will be about the following questions and how they relate: - How many questions do you need to ask a length spectrum to determine it? - Among all surfaces of given genus, how many can be isospectral but not isometric?

The approach to these questions will include finding adapted coordinate sets for moduli spaces and exploring McShane type identities.

ON KHOVANOV HOMOLOGY OF LINKS

MARITHANIA SILVERO CASANOVA
University of Warsaw

Khovanov homology of knots and links was introduced by Mikhail Khovanov at the end of last century. This link invariant, which categorifies Jones polynomial, was nicely reinterpreted by Viro in a purely combinatorial way in terms of Kauffman states. While conceptually simple, this definition becomes impractical when increasing the numbers of crossings of a link diagram.

In this talk we present a new approach to extreme Khovanov homology introduced in [GMS]. With this point of view, we conjecture in [PS] that extreme Khovanov homology is torsion free and prove it for some particular families of links. We also present some advances on the study of torsion in Khovanov homology of torus links.

[GMS]: J. González-Meneses, P. M. G. Manchón and M. Silvero. A geometric description of the extreme Khovanov homology. Proccedings of The Royal Society of Edinburgh: Section A.

[PS]: J. H. Przytycki and M. Silvero. Homotopy type of circle graphs complexes motivated by extreme Khovanov homology. https://arxiv.org/abs/1608.03002

A JONES POLYNOMIAL FOR CURVES ON SURFACES

JULIEN MARCHÉ
Université Pierre et Marie Curie

In a recent work, T. Koberda et R.Santharoubane have studied representations of surface groups with the following property: every simple curve has an image of finite order. These representations come from topological quantum field theory (TQFT) and depend on an integer k called level. I will explain that when k goes to infinity, these representations are controlled by a sort of Jones polynomial which may distinguish simple curves from the others. This is joint work with R. Santharoubane.

CONVEX CO-COMPACT SUBGROUPS OF THE MAPPING CLASS GROUP

KEN BROMBERG
University of Utah

It is a well known question wether there are hyperbolic 4-manifolds that fiber over a surface. For any 4-manifold that fibers of the a surface the fibers will also be a surface (usually of different topological type) and the monodromy defines a homomorphism from the fundamental group of the base surface into the mapping class group of the fiber. Farb and Mosher defined a notion of convex co-compactness for subgroups of the mapping class group and showed that the monodromy map is an injection to a convex co-compact subgroup if the manifold is hyperbolic. It is therefore of great interest to understand convex co-compact subgroups of the mapping class group. Farb and Mosher also showed that any such subgroup is both purely pseudo-Anosove (every non-trivial element is pseudo-Anosov) and undistorted. We will show that that these necessary conditions are also sufficient. This is joint work with M. Bestvina, A. Kent and C. Leininger.

Thematic Session
Mathematics in Space Science

Organized by

James D. Biggs
Politecnico di Milano

Gerard Gómez
Universitat de Barcelona

Josep J. Masdemont
Universitat Politècnica de Catalunya

Schedule

Thursday 10:30 – 11:15, Room Pere Coromines

Bastien Le Bihan

Institut Supérieur de l'Aéronautique et de l'Espace, Toulouse

Semi-analytical Study of the Dynamics about and between the Libration Points of the Sun-Earth-Moon system

Thursday 11:20 – 12:05, Room Pere Coromines

James D. Biggs

Politecnico di Milano

Motion Planning on Lie Groups with Applications to Spacecraft Attitude Guidance and Docking

Thursday 12:10 – 12:55, Room Pere Coromines

Esther Barrabés

Universitat de Girona

Dynamical generation of galactic bridges and tails using a parabolic restricted three-body problem

Friday 10:30 – 11:15, Room Pere Coromines

Elisa Maria Alessi

Italian National Research Council, Rome

Resonant dynamics in the LEO region

Friday 11:20 – 12:05, Room Pere Coromines

Mercè Romero

Universitat de Barcelona

The role of invariant manifolds in galactic dynamics

Friday 12:10 – 12:55, Room Pere Coromines

Camilla Colombo

Politecnico di Milano

Luni-solar perturbations for missions design in highly elliptical orbits

SEMI-ANALYTICAL STUDY OF THE DYNAMICS ABOUT AND BETWEEN THE LIBRATION POINTS OF THE SUN-EARTH-MOON SYSTEM

BASTIEN LE BIHAN

Institut Supérieur de l'Aéronautique et de l'Espace

In this talk, we are interested in consistently computing the natural connections between the libration point L2 of the Earth-Moon system (EML2) and the libration point L1 and L2 of the Sun-(Earth+Moon) system (SEML1,2) in a single periodic and coherent model of the Sun-Earth-Moon system. First, the invariant manifolds are obtained semi-analytically at each point, using the parameterization method, specifically tailored to account for the periodicity of the problem. A systematic search for connection can then be performed in the parameterization space: initial conditions on the center-unstable manifold at EML2 are propagated forward in time and projected on the semi-analytical center manifold at SEML1,2 . A transfer is found each time that the distance of projection is close to zero. These solutions are refined using a differential correction scheme in the parameterization space, which can be coupled with a continuation procedure to easily obtain families of natural continuous transfers. Finally, the resulting trajectories are refined to JPL ephemerides.

MOTION PLANNING ON LIE GROUPS WITH APPLICATIONS TO SPACECRAFT ATTITUDE GUIDANCE AND DOCKING

JAMES D. BIGGS
Politecnico di Milano

This talk will present a general (kinematic) optimal control problem, quadratic in cost, defined on the frame bundle and isometry group of the planar forms that are subject to prescribed boundary conditions on the group. This class of optimal control problem lifts to an integrable Hamiltonian system and the extremals are solved explicitly in terms of Jacobi elliptic functions. A simple structure preserving shooting method is then used to match the boundary conditions at the level of the group. It is then shown how the results of these general optimal control problems can be used in practical motion planning for spacecraft both in applications to attitude guidance and docking.

DYNAMICAL GENERATION OF GALACTIC BRIDGES AND TAILS USING A PARABOLIC RESTRICTED THREE-BODY PROBLEM

ESTHER BARRABÉS
Universitat de Girona

After a close encounter of two galaxies, brigdes and tails can be seen between or around them. A bridge would be an spiral arm between a galaxy and its companion, whereas a tail would correspond to a long and curving set of debris escaping from the galaxy. We consider a parabolic restricted three body problem as a simple model to understand the effect of a close encounter of two galaxies and to give a mechanism that explain the formation of bridges and tails. The parabolic problem consist in the study of the motion of an infinitesimal mass under the gravitational influence of two masses (primaries) moving in two parabolic orbits. Taking into account just one particle, after the close encounter, the particle may stay, or jump to the other galaxy or escape. We will show that the invariant manifolds of the equilibrium points and the heteroclinic connections between explain the formation of bridges and tails.

RESONANT DYNAMICS IN THE LEO REGION

ELISA MARIA ALESSI
Italian National Research Council

The growing awareness of the space debris problem has motivated the scientific community to look for effective and feasible active and passive end-of-life disposal solutions for spacecraft orbiting the Earth, in particular the Low Earth Orbit region (LEO). To this end, it is mandatory to achieve a deep understanding of the dynamics pervading the circumterrestrial space. The talk will address the recent advancements obtained in the framework of the ReDSHIFT H2020 project concerning the dynamical mapping of the LEO region, by means of extensive numerical simulations and analytical studies of the resonances due to solar radiation pressure and third-body perturbations.

THE ROLE OF INVARIANT MANIFOLDS IN GALACTIC DYNAMICS

MERCÈ ROMERO
Universitat de Barcelona

In this talk we will show how invariant manifolds are applied in galactic dynamics, specifically the role they play on the galaxy morphology. Galaxies are not homogeneous distributions of stars and gas, but they present different substructures like bars, spiral arms or rings. They are not even flat distributions. We link these structures and shapes with mathematical invariant objects.

LUNI-SOLAR PERTURBATIONS FOR MISSIONS DESIGN IN HIGHLY ELLIPTICAL ORBITS

CAMILLA COLOMBO
Politecnico di Milano

Highly Elliptical Orbits (HEOs) about the Earth are often selected for astronomy missions, such as INTEGRAL and XMM-Newton, as well as for Earth missions, such as Molniya or Tundra orbits, as they offer vantage point for the observation of the Earth or are used for injection into Geostationary orbit through Geostationary Transfer Orbits. In 2018 the Proba-3 satellites will be injected into a HEO to demonstrate formation flying in the context of a large-scale science mission. HEOs guarantees spending most of the time at an altitude outside the Earth's radiation belt; therefore, long periods of uninterrupted scientific observation are possible. Geo-synchronicity is often opted to meet coverage requirements, enhanced at the apogee, and optimise the ground station down-link. If the inclination is properly selected, HEO can minimise the duration of the motion inside the eclipses. This talk analyses the long-term evolution of spacecraft in HEOs. The dynamics of HEOs with high apogee altitude is mainly influenced by the effect of third body perturbations due to the Moon and the Sun, which induces long-term variations in the eccentricity and the inclination, corresponding to large fluctuations of the orbit perigee. A method is proposed to compute the optimal man oeuvre to perform end-of-life re-entry or transfer into a stable elliptical orbit. The Δ-v manoeuvre is computed in the eccentricity-inclination-anomaly-of-perigee map, first introduced by Kozai. Through these maps, conditions for quasi-frozen, or long-lived libration orbits are identified. In addition, to allow meeting specific mission constraints, stable conditions for quasi-frozen orbits can be selected as graveyard orbits for the end-of-life of HEO missions, such as

the XMM-Newton mission. On the opposite side, unstable conditions can be exploited to target an Earth re-entry at the end-of-mission, such as the INTEGRAL mission.

Thematic Session

Modularity and Diophantine Equations: Exploring Wiles' Universe

Organized by

Luis V. Dieulefait
Universitat de Barcelona

Nuno Freitas
University of British Columbia

Schedule

Thursday 10:30 – 11:15, Room Nicolau d'Olwer

Ana Caraiani

University of Bonn

Galois representations and torsion classes

Thursday 11:20 – 12:05, Room Nicolau d'Olwer

Alain Kraus

Université Pierre et Marie Curie, Paris

Fermat's Last Theorem over some totally real number fields

Thursday 12:10 – 12:55, Room Nicolau d'Olwer

Samuel Le Fourn

École Normale Supérieure de Lyon

Surjectivity of Galois representations of quadratic Q-curves

Friday 10:30 – 11:15, Room Nicolau d'Olwer

Daniel Barrera Salazar

Universitat Politècnica de Catalunya, Barcelona

On the exceptional zeros of p-adic L-functions of Hilbert modular forms

Friday 11:20 – 12:05, Room Nicolau d'Olwer

Jeanine Van Order

Bielefeld University

Bounds for Mordell-Weil ranks via Fourier coecients of automorphic forms on the metaplectic cover of GL2

Friday 12:10 – 12:55, Room Nicolau d'Olwer

Rafael von Känel

Princeton University

Integral points on Hilbert modular varieties

GALOIS REPRESENTATIONS AND TORSION CLASSES

ANA CARAIANI
University of Bonn

I will describe joint work in progress with Allen, Calegari, Gee, Helm, Le Hung, Newton, Scholze, Taylor, and Thorne on potential modularity for elliptic curves over imaginary quadratic fields. The key ingredients are a version of the Taylor-Wiles patching argument due to Calegari and Geraghty, and a result on torsion in the cohomology of Shimura varieties that is joint with Scholze. I will focus on explaining the difficulty in the imaginary quadratic case and how Calegari-Geraghty method comes to the rescue.

FERMAT'S LAST THEOREM OVER SOME TOTALLY REAL NUMBER FIELDS

ALAIN KRAUS
Université Pierre et Marie Curie, Paris

In the past few years, numerous progress have been achieved about the Fermat equation over number fields. We have in particular a criterion, due to Freitas and Siksek, allowing to establish the asymptotic Fermat's Last Theorem over certain totally real number fields. Such is the case for an important proportion of real quadratic fields. In this talk, we are interested in the study of the Fermat equation over totally real number fields, satisfying conditions on the ramification at the prime number 2 and some ray class field. Experimentally, we notice that these assumptions allow often to prove the effective asymptotic Fermat's Last Theorem over such fields. In all of this, we use directly the modular method.

SURJECTIVITY OF GALOIS REPRESENTATIONS OF QUADRATIC Q-CURVES

SAMUEL LE FOURN
École Normale Supérieure de Lyon

One initial tool in Wiles' proof of Fermat's last theorem is using the Galois representation on the p-torsion of an elliptic curve and proving that this representation is irreducible except for p very small (with an absolute bound). This was obtained by Mazur in 1977, and we can more generally ask if this representation is surjective for large enough p (with an absolute bound again). Surprisingly, we still don't know the answer to this question (called "Serre's uniformity problem") for elliptic curves over Q without CM, because of a particulary resistant part of the problem. In this talk, I will study quadratic imaginary Q-curves, which are "almost" elliptic curves over Q, and prove that for (explicit) large enough p, the representation is surjective. I will explain how the proof works, and why we bypass the resistant part in this specific situation. If time allows it, I will also briefly describe how to use these Q-curves to solve certains diophantine equations.

ON THE EXCEPTIONAL ZEROS OF P-ADIC L-FUNCTIONS OF HILBERT MODULAR FORMS

DANIEL BARRERA SALAZAR
Universitat Politècnica de Catalunya, Barcelona

The use of modular symbols to attach p-adic L-functions to Hecke eigenforms goes back to the work of Manin et al in the 70s. In the 90s, Stevens developed his theory of overconvergent modular symbols, which was successfully used to construct p-adic L-functions on the eigenvariety. In this talk we will present a work in collaboration with Mladen Dimitrov and Andrei Jorza in which we generalize this approach to the Hilbert modular setting and prove new instances of the exceptional zero conjecture.

BOUNDS FOR MORDELL-WEIL RANKS VIA FOURIER COEFFICIENTS OF AUTOMORPHIC FORMS ON THE METAPLECTIC COVER OF GL_2.

JEANINE VAN ORDER
Bielefeld University

Let E be an elliptic curve of conductor N defined over the rationals, K a quadratic extension of absolute discriminant D, and e the sign of the Hasse-Weil L-function of E over K. Given an integer c prime to N, assume there exists a ring class extension $K[c]$ of conductor c over K (as is always the case if K is imaginary). If $e = 1$ and ND is sufficiently large, then I will explain how to use bounds for the Fourier coefficients of automorphic forms on the metaplectic cover of GL_2 to show that the Mordell-Weil group $E(K[c])$ has rank zero, and in fact is trivial if K is real quadratic. (The latter setting is not accessible by Heegner point or Euler system techniques, and relies on recent work of Darmon-Rotger). The strategy here is to estimate average central values of self-dual Rankin-Selberg and triple product L-functions, and can be used to derive a more general class of results in the setting of CM fields, as I will also explain.

INTEGRAL POINTS ON HILBERT MODULAR VARIETIES

RAFAEL VON KÄNEL
Princeton University

We will discuss a joint project with Arno Kret in which we studied integral points on Hilbert modular varieties. In particular we shall present explicit upper bounds for the height and number of integral points on Hilbert modular varieties. We shall also explain the strategy of proof.

Thematic Session

New Perspectives in PDEs and Applications

Organized by

Xavier Cabré
ICREA and Universitat Politècnica de Catalunya

Enrico Valdinoci
Weierstrass Institute for Applied Analysis and Stochastics

Schedule

Thursday 10:30 – 11:15, Room Pi i Sunyer

Benoit Perthame

Université Paris 6

Models for tumor growth and the Hele-Shaw asymptotic

Thursday 11:20 – 12:05, Room Pi i Sunyer

Joaquim Serra

Swiss Federal Institute of Technology in Zürich

Nonlocal obstacle problems: regularity of the solutions and the free boundaries

Thursday 12:10 – 12:55, Room Pi i Sunyer

Cyril Imbert

Centre National de la Recherche Scientifique and École Normale Supérieure, Paris

Weak Harnack inequality for the Boltzmann equation without cut-off

Friday 10:30 – 11:15, Room Pi i Sunyer

Alberto Farina

Université Picardie, Amiens

A Bernstein-Type Result for the Minimal Surface Equation

Friday 11:20 – 12:05, Room Pi i Sunyer

Guido De Philippis

Scuola Internazionale Superiore di Studi Avanzati, Triestre

On the structure of A-free measures and applications

Friday 12:10 – 12:55, Room Pi i Sunyer

Enrico Valdinoci

Weierstrass Institute for Applied Analysis and Stochastic

Long-range phase transitions and minimal surfaces

MODELS FOR TUMOR GROWTH AND THE HELE-SHAW ASYMPTOTIC

BENOIT PERTHAME
Université Paris 6

The growth of solid tumors can be described at a number of different scales from the cell to the organ scales. For a large number of cells, the 'fluid mechanical' approach has been advocated recently by many authors in mathematics or biophysics. Several levels of mathematical descriptions are commonly used, including possibly elasticity, viscoelastic laws, nutrients, active movement, surrounding tissue, and several other features.

We will focuss on the links between two types of mathematical models. The 'microscopic' or 'compressible' description is at the cell population density level and a more macroscopic, description is based on a free boundary problem close to the classical Hele-Shaw equation. Asymptotic analysis is a tool to derive these Hele-Shaw free boundary problems from cell density systems in the stiff pressure limit. This modeling also opens other questions as circumstances in which instabilities may develop or the role of necrotic cores.

This work is a collaboration with F. Quiros and J.-L. Vazquez (Universidad Autonoma Madrid), A. Mellet, M. Tang (SJTU) and N. Vauchelet (LJLL).

NONLOCAL OBSTACLE PROBLEMS: REGULARITY OF THE SOLUTIONS AND THE FREE BOUNDARIES

JOAQUIM SERRA
Swiss Federal Institute of Technology in Zürich

We will introduce some recent results in collaboration with L. Caffarelli and X. Ros-Oton on the optimal regularity of the solutions and the regularity of the free boundaries (near regular points) for nonlocal obstacle problems. The main novelty is that we obtain results for different operators than the fractional Laplacian. Indeed, we can consider infinitesimal generators of non rotationally invariant stable Lévy processes.

WEAK HARNACK INEQUALITY FOR THE BOLTZMANN EQUATION WITHOUT CUT-OFF

CYRIL IMBERT

Centre National de la Recherche Scientifique and École Normale Supérieure, Paris

In this talk we present a weak Harnack inequality and Hölder estimates for a large class of kinetic integro-differential equations. We explain that the Boltzmann equation without cut-off can be written in this form and satisfies our assumptions provided that the mass density is bounded away from vacuum and mass, energy and entropy densities are bounded above. As a consequence, we explain how to derive a local Hölder estimate and a quantitative lower bound for solutions of the (inhomogeneous) Boltzmann equation without cut-off.

A BERNSTEIN-TYPE RESULT FOR THE MINIMAL SURFACE EQUATION

ALBERTO FARINA

Laboratoire Amiénois de Mathématique Fondamentale et Appliquée, CNRS

We prove the following Bernstein-type theorem: *if u is a solution to the minimal surface equation over $\mathbb{R}^N$, such that $N-1$ partial derivatives $\frac{\partial u}{\partial x_j}$ are bounded on one side (not necessarily the same), then u is an affine function.* Besides its novelty, our theorem also provides a new, simple and self-contained proof of celebrated results of Moser and of Bombieri & Giusti.

ON THE STRUCTURE OF $\mathcal{A}$-FREE MEASURES AND APPLICATIONS

GUIDO DE PHILIPPIS
Scuola Normale Superiore, Pisa

I will show a general structure theorem for the singular part of $\mathcal{A}$-free Radon measures, where $\mathcal{A}$ is a linear PDE operator. By applying the theorem to suitably chosen differential operators $\mathcal{A}$,one can obtain a simple proof of Alberti's rank-one theorem and its extensions to functions of bounded deformation (BD). I will also show some consequences concerning the sharpness of Rademacher Theorem and the structure of Ambrosio–Kirchheim top-dimensional metric current in $\mathbb{R}^d$.

LONG-RANGE PHASE TRANSITIONS AND MINIMAL SURFACES

ENRICO VALDINOCI

Weierstrass Institute for Applied Analysis and Stochastic

We discuss some phase transition model with particle interactions modeled by a kernel with polynomial decay. At a large scale, we relate the interfaces of this model with either classical minimal surfaces or nonlocal ones, depending on the decay of the interaction. We give rigidity, symmetry and regularity results about these objects and present related geometric flows.

Thematic Session

Progress in Transport Phenomena

Organized by

Joan Orobitg
Universitat Autònoma de Barcelona

Filippo Santambrogio
Université Paris-Sud

Schedule

Thursday 16:30 – 17:15, Room Pi i Sunyer

Mark Peletier

Technische Universiteit Eindhoven

Upscaling the dynamics of dislocations

Thursday 17:20 – 18:05, Room Pi i Sunyer

Marie-Therese Wolfram

University of Warwick

Analysis of a cross-diffusion model with excluded volume effects and asymptotic gradient flows

Thursday 18:10 – 12:55, Room Pi i Sunyer

Juan Soler

Universidad de Granada

Exploring new solutions to the incompressible Euler equations

Friday 16:30 – 17:15, Room Pi i Sunyer

Filippo Santambrogio

Université Paris-Sud

Congested transport problems and equilibria

Friday 17:20 – 18:05, Room Pi i Sunyer

Paola Goatin

Institut National de Recherche en Informatique et en Automatique, Sophia Antipolis

Non-local conservation laws arising in traffic modeling

Friday 18:10 – 18:55, Room Pi i Sunyer

José A. Carrillo

Imperial College London

Minimizing interaction energies

UPSCALING THE DYNAMICS OF DISLOCATIONS

MARK PELETIER
Technische Universiteit Eindhoven

Plasticity, the permanent deformation that one observes in metals, is the net effect of the movement of a large number of microscopic defects in the atomic lattice. These defects, called dislocations, are curve-like topological mismatches, and migrate through the metal under the influence of internal and external forces. Macroscopic, permanent, deformation arises through the concerted movement of a large number of these dislocations. It is a major challenge to connect a microscopic description of dislocation movement on one hand with models of macroscopic plastic behaviour on the other hand. If this were possible, then much could be gained: metals could be designed at the workstation with tailor-made properties, design of hybrid materials would become much easier, and generally the holy grail of 'materials by design' would come a little closer. At this stage we are not able to do this; there is a major gap between the models at these different spatial and temporal scales. Part of the difficulty lies in the complex interactions between dislocations: they attract and repel each other, and form complex higher-level structures that appear to play an important role in determining the macroscopic behaviour. Interestingly, the situation for the dynamics of the dislocations is significantly more complex than that of the energetics.

I will outline some recent results in this field, describe some of our own recent results in two dimensions, and mention some open questions and one or two mysteries.

ANALYSIS OF A CROSS-DIFFUSION MODEL WITH EXCLUDED VOLUME EFFECTS AND ASYMPTOTIC GRADIENT FLOWS

MARIE-THERESE WOLFRAM
University of Warwick

In this talk we discuss the analysis of a cross-diffusion PDE system for a mixture of hard spheres, which was derived by Bruna and Chapman from a stochastic system of N interacting Brownian particles using the method of matched asympototics. Gradient-flow techniques have become a well established tool to study these kind of nonlinear PDEs. Hence expressing a nonlinear diffusion equation as a gradient flow of an entropy is a very desirable feature. The PDE system under consideration satisfies a gradient flow structure if particles have the same size. For particles of different size we can interpret the equations as an asymptotic gradient flow structure (which results from the asymptotic expansion in the derivation). We shall use this asymptotic gradient flow structure to provide existence of stationary solutions and stability close to equilibrium. Furthermore we discuss global in time existence for the full gradient flow system and illustrate the behavior of the model with various numerical simulations.

Joint work with M. Bruna (University of Oxford), M. Burger (WWU Münster) and Helene Ranetbauer (RICAM)

EXPLORING NEW SOLUTIONS TO THE INCOMPRESSIBLE EULER EQUATIONS

JUAN SOLER
Universidad de Granada

The idea of this talk is to introduce some special solutions of the incompressible Euler equation through the connection with other systems of interest such as the Schrödinger, Helmholtz or Allen-Cahn equations, by means of appropriate changes of coordinates and bifurcation theory.

CONGESTED TRANSPORT PROBLEMS AND EQUILIBRIA

FILIPPO SANTAMBROGIO
Université Paris-Sud

The talk will be a survey about the continuous models that we introduced years ago with Carlier and Jimenez to provide a continuous formulation of the so-called Wardrop equilibrium problem. This problem, in its original formulation, is concerned with the traffic intensity on a given network, where every agent must choose his own trajectory so as to minimize his cost, but this cost depends on the traffic intensity he meets on his road, i.e. on the choices of all the other agents. This equilibrium problem is of variational origin, as the equilibrium can be obtained by minimizing a global congestion cost. The continuous model provides a description of the same phenomenon, but replacing the finite network with a domain in R^n where all trajectories are admissible. The traffic intensity turns out to be ruled in some cases by an elliptic PDE, which is very degenerate in all situations which are relevant in modeling, and obtained by a convex minimization problem for which efficient algorithms exist. In the talk, I will give an introduction to these questions, and present some of the most recent results and open questions (based on joint works with Carlier, Jimenez, Brasco, Benmansour, Peyré, and Vespri, and on the contributions by many other people).

NON-LOCAL CONSERVATION LAWS ARISING IN TRAFFIC MODELING

PAOLA GOATIN

Institut National de Recherche en Informatique et en Automatique, Sophia Antipolis

Equations with non-local flux have been recently introduced in traffic flow modeling to account for the reaction of drivers or pedestrians to the surrounding density of other individuals. While pedestrians are likely to react to the presence of people all around them, drivers will mainly adapt their velocity to the downstream traffic, assigning a greater importance to closer vehicles. In particular, and in contrast to classical (without integral terms) macroscopic equations, these models are able to display finite acceleration of vehicles through Lipschitz bounds on the mean velocity and lane formation in crossing pedestrian flows. We will also present recent results on micro-macro limits of empirical measures converging to measure-valued solutions of the corresponding macroscopic evolution equation.

MINIMIZING INTERACTION ENERGIES

JOSÉ A. CARRILLO
Imperial College London

I will start by reviewing some recent results on qualitative properties of local minimizers of the interaction energies to motivate the main topic of my talk: to discuss global minimizers. We will show the existence of compactly supported global minimizes under quite mild assumptions on the potential in the complementary set of classical H-stability in statistical mechanics. A strong connection with the classical obstacle problem appears very useful when the singularity is strong enough at zero. I will also discuss recent results in the theory of dislocations where two dimensional densities are reduced to one dimensional densities for non radially symmetric potentials.