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Surjectivity of Galois representations of quadratic Q-curves

One initial tool in Wiles' proof of Fermat's last theorem is using the Galois representation on the p-torsion of an elliptic curve and proving that this representation is irreducible except for p very small (with an absolute bound). This was obtained by Mazur in 1977, and we can more generally ask if this representation is surjective for large enough p (with an absolute bound again). Surprisingly, we still don't know the answer to this question (called "Serre's uniformity problem") for elliptic curves over Q without CM, because of a particulary resistant part of the problem. In this talk, I will study quadratic imaginary Q-curves, which are "almost" elliptic curves over Q, and prove that for (explicit) large enough p, the representation is surjective. I will explain how the proof works, and why we bypass the resistant part in this specific situation. If time allows it, I will also briefly describe how to use these Q-curves to solve certains diophantine equations.